TEXAS STATE UNIVERSITY

The rising STAR *of Texas*

Differential Equations and Applied Math Seminar

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12-1pm March 25th, 2022

Zoom and 330 DH

Title: Electrostatic partners and zeros of orthogonal and multiple orthogonal polynomials

Abstract: The well-known electrostatic interpretation of the zeros of Hermite, Laguerre or Jacobi polynomials, which goes back to the 1885 work of Stieltjes, is one of the most popular models in the theory of orthogonal polynomials. It was picked up and extended to several contexts, such as orthogonal and quasi-orthogonal polynomials on the real line and the unit circle, for classical and semiclassical weights. Our first goal is to generalize the known electrostatic interpretations.

For a given polynomial P with simple zeros, and a given semiclassical weight w, we present a construction that yields a linear second order differential equation (ODE), and in consequence, an electrostatic model for zeros of P. The coefficients of this ODE are written in terms of a dual polynomial that we call the electrostatic partner of P. This construction is absolutely general and can be carried out for any polynomial with simple zeros and any semiclassical weight on the complex plane. An additional assumption of quasi-orthogonality of P with respect to w allows us to give more precise bounds on the degree of the electrostatic partner. In the case of orthogonal and quasi-orthogonal polynomials, we recover some of the known results and

generalize others.

For the Hermite–Padé or multiple orthogonal polynomials of type II, this approach yields a system of linear second-order differential equations, from which we derive an electrostatic interpretation of their zeros in terms of a vector equilibrium (something that was unknown). More detailed results are obtained in the special cases of Angelesco, Nikishin, and generalized Nikishin systems. We also discuss the discreteto-continuous transition of these models in the asymptotic regime, as the number of zeros tends to infinity, into the known vector equilibrium problems. If time permits, we will discuss how the system of obtained second-order ODEs yields a third-order differential equation for these polynomials, well described in the literature, as well as present several illustrative examples.

This is a joint work with R. Orive (Universidad de La Laguna, Canary Islands, Spain) and J. Sanchez-Lara (Granada University, Spain).

Interested faculty, graduate and undergraduate students are encouraged to attend.